AD-A077 137

WISCONSIN UNIV-MADISON MATHEMATICS RESEARCH CENTER

NONLINEAR SCHROEDINGER EVOLUTION EQUATIONS. (U)

SEP 79 H BREZIS , T GALLOUET

UNCLASSIFIED

MRC-TSR-1992

END

ANTE

ANT

I EVEW (11) to

MRC Technical Summary Report #1992

NONLINEAR SCHRÖDINGER EVOLUTION EQUATIONS

H. Brezis and T. Gallouet

Mathematics Research Center University of Wisconsin—Madison 610 Walnut Street Madison, Wisconsin 53706

otember 1979

AD A O 7713

Received July 30, 1979)

Approved for public release Distribution unlimited

Sponsored by

U. S. Army Research Office P. O. Box 12211 Research Triangle Park North Carolina 27709 UNIVERSITY OF WISCONSIN-MADISON
MATHEMATICS RESEARCH CENTER

NONLINEAR SCHRÖDINGER EVOLUTION EQUATIONS

H. Brezis 1 and T. Gallouet

Technical Summary Report #1992 September 1979

ABSTRACT

We consider the nonlinear Schrödinger equation

$$\begin{cases} i \frac{\partial u}{\partial t} - \Delta u + k |u|^2 u = 0 & \text{in} & \Omega \times [0, \infty) \\ u(x,t) = 0 & \text{in} & \partial \Omega \times [0, \infty) \\ u(x,0) = u_0(x) & \text{in} & \Omega \end{cases}$$

where Ω is a bounded domain or an exterior domain of \mathbf{R}^2 . Such an equation has been extensively studied when $\Omega = \mathbf{R}^2$, but the methods do not apply if $\Omega \neq \mathbf{R}^2$. We prove that there exists a unique global smooth solution if $k \geq 0$ or if k < 0 and $|k| \int |u_0|^2 < 4$. The proof relies on a new interpolation-embedding inequality:

$$\|\mathbf{u}\|_{\mathbf{L}^{\infty}} \leq C[1 + \sqrt{\log(1 + \|\mathbf{u}\|_{2})}]$$
 for every $\mathbf{u} \in \mathbb{H}^{2}$ with $\|\mathbf{u}\|_{\mathbf{H}^{1}} \leq 1$.

AMS (MOS) Subject Classifications: 47H15, 46E35, 35K55, 35L60, 45E30

Key Words: Nonlinear Schrodinger equation, Global solutions, Sobolev embedding, Interpolation inequality

Work Unit Number 1 (Applied Analysis)

Dept. de Mathématiques, Université Paris VI, 4, pl. Jussieu, 75230 Paris Cedex 05, France.

Sponsored by the United States Army under Contract No. DAAG29-75-C-0024.

SIGNIFICANCE AND EXPLANATION

The nonlinear Schrödinger equation occurs in the study of some problems in nonlinear optics (propagation of laser beams through the atmosphere or in a plasma). It has been considered by many authors. The main novelty of the present result is that it applies to the propagation of beams in channels (instead of the whole space). The proof relies on a new Sobolev-Orlicz embedding inequality which could be useful in other situations.

Access	on For	
NTIS (InikI	
DDC TA	3	
Unanno	inced	
Justif	ication	
Ву		
	bution/	
Avail	ability	Codes
	Availa	
Dist.	speci	
1		
A		

The responsibility for the wording and views expressed in this descriptive summary lies with MRC, and not with the authors of this report.

NONLINEAR SCHRODINGER EVOLUTION EQUATIONS

H. Brezis 1 and T. Gallouet

Let Ω be a domain in R^2 with compact smooth boundary Γ (Ω could be for example a bounded domain or an exterior domain). Consider the equation

(1)
$$\begin{cases} i \frac{\partial u}{\partial t} - \Delta u + k|u|^2 u = 0 & \text{in } \Omega \times [0, \infty) \\ u(x,t) = 0 & \text{in } \Gamma \times [0, \infty) \\ u(x,0) = u_0(x) \end{cases}$$

where u(x,t) is a complex valued function and $k \in \mathbf{R}$ is a constant. Problem (1) which occurs in nonlinear optics when $\Omega = \mathbf{R}^2$ has been extensively studied in this case (see [1],[2],[3],[5],[8]), but we are not aware of any known result when $\Omega \neq \mathbf{R}^2$.

Our main result is the following:

Theorem 1. Let $u_0 \in H^2(\Omega) \cap H_0^1(\Omega)$. Assume that <u>one</u> of the following conditions holds

- (a) either $k \ge 0$,
- (b) or k < 0 and $|k| \int |u_0(x)|^2 dx < 4$.

Then there exists a unique solution of (1) such that

$$u \in C([0,\infty); H^2(\Omega)) \cap C^1([0,\infty); L^2(\Omega))$$
.

The proof of Theorem 1 relies on several Lemmas. The first Lemma is of interest for its own sake; it is a new interpolation-embedding inequality.

¹Dept. de Mathématiques, Université Paris VI, 4, pl. Jussieu, 75230 Paris Cedex 05, France.

Sponsored by the United States Army under Contract No. DAAG29-75-C-0024.

In what follows we denote by C various constants depending only on $\,\Omega_{\star}\,$ Lemma 2. We have

(2)
$$\|\mathbf{u}\|_{\infty} \le C(1 + \sqrt{\log(1 + \|\mathbf{u}\|_{2})})$$

for every $u \in H^2(\Omega)$ with $\|u\|_{H^1} \leq 1$.

<u>Proof.</u> It is well known that an H^2 function on Ω can be extended by an H^2 function on R^2 . More precisely one can construct an extension operator P such that:

P is a bounded operator from $H^{1}(\Omega)$ into $H^{1}(R^{2})$

P is a bounded operator from $H^2(\Omega)$ into $H^2(R^2)$

 $Pu_{|\Omega} = u$ for every $u \in H^{1}(\Omega)$.

Let $u \in H^2(\Omega)$ with $\|u\|_{H^1} \le 1$. Let v = Pu and denote by \hat{v} the Fourier transform of v. We clearly have

(3)
$$\|(1 + |\xi|) \hat{v}\|_{L^{2}(\mathbb{R}^{2})} \leq C$$

(4)
$$\|(1 + |\xi|^2) \hat{\mathbf{v}}\|_{\mathbf{L}^2(\mathbf{R}^2)} \leq C \|\mathbf{u}\|_{\mathbf{H}^2(\Omega)}$$

(5)
$$\|\mathbf{u}\|_{\mathbf{L}^{\infty}(\Omega)} \leq \|\mathbf{v}\|_{\mathbf{L}^{\infty}(\mathbf{R}^{2})} \leq C\|\hat{\mathbf{v}}\|_{\mathbf{L}^{1}(\mathbf{R}^{2})}.$$

For R > 0 we write

$$\|\hat{\mathbf{v}}\|_{\mathbf{L}^{1}} = \begin{cases} |\hat{\mathbf{v}}(\xi)| \, \mathrm{d}\xi + \int_{|\xi| \ge R} |\hat{\mathbf{v}}(\xi)| \, \mathrm{d}\xi = \int_{|\xi| < R} (1 + |\xi|) |\hat{\mathbf{v}}(\xi)| \frac{1}{1 + |\xi|} \, \mathrm{d}\xi \end{cases}$$

$$+ \int_{|\xi| \ge R} (1 + |\xi|^{2}) |\hat{\mathbf{v}}(\xi)| \frac{1}{1 + |\xi|^{2}} \, \mathrm{d}\xi$$

$$\leq C \left[\int_{|\xi| < R} \frac{1}{(1 + |\xi|)^{2}} \, \mathrm{d}\xi \right]^{1/2} + C \|\mathbf{u}\|_{\mathbf{H}^{2}} \left[\int_{|\xi| \ge R} \frac{1}{(1 + |\xi|^{2})^{2}} \, \mathrm{d}\xi \right]^{1/2}$$

by Cauchy-Schwarz, (3) and (4). A straightforward computation leads to

$$\|\hat{\mathbf{v}}\|_{L^{1}} \le C[\log(1+R)]^{1/2} + C\|\mathbf{u}\|_{H^{2}}(1+R)^{-1}$$

by every $R \ge 0$. We obtain (2) by choosing $R = \|u\|_{H^2}$.

Lemma 3. We have

(6)
$$\|\|u\|^2 u\|_{\dot{H}^2} \leq C \|u\|^2 \|u\|_{\dot{H}^2} \quad \text{for every } u \in \dot{H}^2(\Omega) .$$

Proof of Lemma 3. Let D denote any first order differential operator. For $u \in H^2$ we have

$$|D^{2}(|u|^{2}u)| \leq C(|u|^{2}|D^{2}u| + |u| |Du|^{2})$$
,

and so

On the other hand an inequality of Gagliardo-Nirenberg (see [6]) implies that

(8)
$$\|\mathbf{u}\|_{\mathbf{W}^{1,4}} \leq C \|\mathbf{u}\|_{\mathbf{L}^{\infty}}^{1/2} \|\mathbf{u}\|_{\mathbf{H}^{2}}^{1/2} .$$

Combining (7) and (8) we obtain (6).

Finally we recall the following well known result essentially due to Segal [7]:

Lemma 4. Assume H is a Hilbert space and A: $D(A) \subseteq H + H$ is an m-accretive linear operator. Assume F is a mapping from D(A) into itself which is Lipschitz on every bounded set of D(A). Then for every $u_0 \in D(A)$, there exists a unique solution u of the equation

$$\begin{cases} \frac{d\mathbf{u}}{d\mathbf{t}} + \mathbf{A}\mathbf{u} = \mathbf{F}\mathbf{u} \\ \mathbf{u}(0) = \mathbf{u}_0 \end{cases}$$

defined for t ϵ [0,T_{max}) such that

$$u \in C^{1}([0,T_{max}); H) \cap C([0,T_{max}); D(A))$$

with the additional property that

$$\begin{cases} \text{ either } T_{\text{max}} = \infty \\ \text{ or } T_{\text{max}} < \infty \text{ and } \lim_{t \to T_{\text{max}}} \text{ fu}(t) \text{ if } + \text{ if } \text{Au}(t) \text{ if } = \infty. \end{cases}$$

Proof of Theorem 1. We apply Lemma 4 in $H = L^2(\Omega)$ to $Au = i\Delta u$,

 $D(A) = H^2(\Omega) \cap H_0^1(\Omega), \text{ Fu = ik|u|}^2 \text{u. We shall show that } T_{max} = \infty \text{ by proving}$ that $\|u(t)\|_{H^2}$ remains bounded on every finite time interval.

First we multiply (1) by \bar{u} and consider the imaginary part. This leads to

(9)
$$u(t) = u_0$$
.

Next we multiply (1) by $\frac{\partial \overline{u}}{\partial t}$ and consider the real part. This leads to (10) $\frac{1}{2} \int |\nabla u(x,t)|^2 dx + \frac{k}{4} \int |u(x,t)|^4 dx \equiv E_0$

where

$$E_0 = \frac{1}{2} \int_{\Omega} |W_0(x)|^2 dx + \frac{k}{4} \int_{\Omega} |u_0(x)|^4 dx$$
.

We claim that $\|u(t)\|$ remains bounded for t>0. Indeed, this is clear when $k\geq 0$. While if k<0 we have

(11)
$$\int |\nabla u(x,t)|^2 \le \frac{|k|}{2} \int |u(x,t)|^4 dx + 2 E_0.$$

On the other hand an inequality of Gagliardo and Nirenberg ([6]) shows that (1)

⁽¹⁾ In order to obtain the constant 1/2 one proceeds as follows. For $\varphi \in C_0^\infty(\mathbb{R}^2) \quad \text{we have} \quad |\varphi\left(\mathbf{x}_1,\mathbf{x}_2\right)| \leq \frac{1}{2} \int\limits_{-\infty}^{+\infty} |\varphi_{\mathbf{x}_1}(\mathbf{t},\mathbf{x}_2)| \, \mathrm{d}\mathbf{t}, \\ |\varphi\left(\mathbf{x}_1,\mathbf{x}_2\right)| \leq \frac{1}{2} \int\limits_{-\infty}^{+\infty} |\varphi_{\mathbf{x}_2}(\mathbf{x}_1,\mathbf{s})| \, \mathrm{d}\mathbf{s}. \quad \text{Thus} \quad \int\limits_{\mathbb{R}^2} |\varphi|^2 \, \mathrm{d}\mathbf{x} \leq \frac{1}{4} \int\limits_{\mathbb{R}^2} |\varphi_{\mathbf{x}_1}| \, \mathrm{d}\mathbf{x} \int\limits_{\mathbb{R}^2} |\varphi_{\mathbf{x}_2}| \, \mathrm{d}\mathbf{x}.$ Choosing $\varphi = |\mathbf{u}|^2$ leads to $\int |\mathbf{u}|^4 \, \mathrm{d}\mathbf{x} \leq \int |\mathbf{u}|^2 \, \mathrm{d}\mathbf{x} (\int\limits_{\mathbf{x}_1} |\mathbf{u}_{\mathbf{x}_2}|^2 \, \mathrm{d}\mathbf{x})^{1/2} (\int\limits_{\mathbb{R}^2} |\mathbf{u}_{\mathbf{x}_2}|^2 \, \mathrm{d}\mathbf{x})^{1/2} \leq \frac{1}{2} \int\limits_{\mathbb{R}^2} |\mathbf{u}|^2 \, \mathrm{d}\mathbf{x} \int\limits_{\mathbb{R}^2} |\nabla \mathbf{u}|^2 \, \mathrm{d}\mathbf{x}.$

(12)
$$\int |\mathbf{u}|^4 d\mathbf{x} \leq \frac{1}{2} \int |\mathbf{u}|^2 d\mathbf{x} \int |\nabla \mathbf{u}|^2 d\mathbf{x}$$
$$= \frac{1}{2} \int |\mathbf{u}_0|^2 d\mathbf{x} \int |\nabla \mathbf{u}|^2 d\mathbf{x} .$$

Combining (11), (12) and assumption (b) in Theorem 1 we see that

(13)
$$\|\mathbf{u}(t)\|_{\dot{H}^1} \leq C$$

where C is independent of t.

We now denote by S(t) the L^2 isometry group generated by -A. From (1) we have

$$u(t) = S(t)u_0 + ik \int_0^t S(t - s)|u(s)|^2 u(s)ds$$

and so

$$Au(t) = S(t)Au_0 + ik \int_0^t S(t - s)A[|u(s)|^2u(s)]ds$$
.

Thus

(14)
$$\|Au(t)\|_{L^{2}} \le \|Au_{0}\|_{L^{2}} + |k| \int_{0}^{t} \|A[|u(s)|^{2}u(s)]\|_{L^{2}} ds$$
.

Lemma 3 implies that

$$\|A[|u(s)|^2u(s)]\|_{L^2} \le C\|u(s)\|_{L^\infty}^2 \|u(s)\|_{H^2}.$$

From Lemma 2 and estimate (13) we deduce that

$$\|u(s)\|_{L^{\infty}} \le C(1 + \sqrt{\log(1 + \|u(s)\|_{H^2})})$$
.

Hence (14) leads to

(15)
$$\|\mathbf{u}(t)\|_{\dot{H}^2} \le c + c \int_0^t \|\mathbf{u}(s)\|_{\dot{H}^2} [1 + \log(1 + \|\mathbf{u}(s)\|_{\dot{H}^2})] ds$$
.

We denote by G(t) the RHS in (15); thus

$$G'(t) = C||u(t)||_{H^2}[1 + log(1 + ||u(t)||_{H^2})] \le CG(t)[1 + log(1 + G(t))]$$
.

Consequently

$$\frac{d}{dt} \log[1 + \log(1 + G(t))] \le C$$

and we find an estimate for $\|u(t)\|_{H^2}$ of the form

$$\|\mathbf{u}(t)\|_{H^2} \leq e^{\alpha e^{\beta t}}$$

for some constants α and β . Therefore $\|u(t)\|_{H^2}$ remains bounded on every finite time interval and so we must have $T_{max} = \infty$.

Remarks. 1) The proof of Theorem 1 leads to an estimate of the form $\|u(t)\|_{\infty} \leq \alpha e^{\beta t} \ . \ \ \text{We do not know whether} \ \ \|u(t)\|_{\infty} \ \ \text{remains actually bounded as} \ \ t + \infty.$

2) When k < 0 and $|\mathbf{k}| \int |\mathbf{u}_0|^2 > 4$, it is known (see [4],[2]) if $\Omega = \mathbb{R}^2$ that the solution of (1) corresponding to some initial conditions may blow up in finite time. A similar phenomenon presumably occurs when $\Omega \neq \mathbb{R}^2$.

REFERENCES

- [1] J. B. Baillon, T. Cazenave, and M. Figueira, Equation de Schrödinger nonlinéaire, C. R. Acad. Sc. Paris 284 (1977), p. 869-872.
- [2] T. Cazenave, Equations de Schrödinger nonlineaires,
- [3] J. Ginibre and G. Velo, On a class of nonlinear Schrödinger equations,
- [4] R. T. Glassey, On the blowing up of solutions to the Cauchy problem for the nonlinear Schrödinger equation, J. Math. Phys. 18 (1977), p. 1794-1979.
- [5] J. E. Lin and W. A. Strauss, Decay and scattering of solutions of a nonlinear Schrödinger equation, J. Funct. Anal. 30 (1978), p. 245-263.
- [6] L. Nirenberg, On elliptic partial differential equations, Ann. Sc. Norm. Sup. Pisa 13 (1959), p. 115-162.
- [7] I. Segal, Nonlinear semi-groups, Ann. of Math. 78 (1963), p. 339-364.
- [8] W. A. Strauss, The nonlinear Schrödinger equation, in Contemporary

 Developments in Continuum Mechanics and PDE, G. de la Penha and

 L. Medeiros ed., North Holland (1978), p. 452-465.

HB/TG/scr

DAAG29-75-C-9024 Deficient Submits DAAG29-75-C-9024 DAAG29-75-C	REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
S. TYPE OF REPORT & PERIOD COVERED Summary Report - no specification of the specification of	1. REPORT NUMBER	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
Summary Report - no specification period Performing period Performing organization name and address Mathematics Research Center, University of Sid Walnut Street Madison, Wisconsin 53706 I. Controlling office name and address U. S. Army Research Office P. O. Box 1221 Research Triangle Park, North Carolina 27709 Monitoring Gency name a address; different from Controlling Office Distribution statement (of this Report) Approved for public release; distribution unlimited. Distribution Statement (of the abstract entered in Black 20, If different from Report) Research Triangle Park, North Carolina 2709 Approved for public release; distribution unlimited. Distribution Statement (of the abstract entered in Black 20, If different from Report) Research Triangle Park (of the abstract entered in Black 20, If different from Report) Distribution Statement (of the abstract entered in Black 20, If different from Report) Research Triangle Park (of the abstract entered in Black 20, If different from Report) Research Triangle Park (of the abstract entered in Black 20, If different from Report) Suppose the provided park of the suppose of the provided park of the park of the provided park of the park o	1992		
Summary Report - no specification period (a performing period (b performing order) to performing order) I. Brezis and T. Gallouet Performing orderization name and address (b) DAAG29-75-C-0024 I. Brezis and T. Gallouet DAAG29-75-C-0024 DAAG29-75-C-0024 I. Prooffant Element, Project, Task And Address (b) Wisconsin (b) Walnut Street (b) Wisconsin (b) Wis	TITLE (and Subtitle)		5. TYPE OF REPORT & PERIOD COVERED
6. PERFORMING ORG. REPORT NUMBER (a) 1. Brezis and T./Gallouet 1. Brezis and T./Gallouet 2. Performing organization name and address Mathematics Research Center, University of SiO Walnut Street 3. Contract or grant number 1 - Applied Analysis 3. S. Army Research Office 3. S. Army Research Office 4. Controlling office Name and address 5. S. Army Research Office 6. Do Box 12211 7. DISTRIBUTION STATEMENT (of this Report) 8. Supplementary notes 9. Key words (Continue on severes side if necessary and identify by block number) 9. Abstract (Continue on severes side if necessary and identify by block number) 9. Abstract (Continue on severes side if necessary and identify by block number) 10. Distribution in the number of the number	2		Summary Report - no specific
DAAG29-75-C-0024 Performing Organization name and address Mathematics Research Center, University of Molitowal Street Madison, Wisconsin 53706 I. Controlling Office name and Address U. S. Army Research Office P.O. Box 12211 Research Triangle Park, North Carolina 27709 A MONITORING IGENCY NAME & ADDRESS(If different from Controlling Office) DISTRIBUTION STATEMENT (of this Report) DISTRIBUTION STATEMENT (of this abstract entered in Black 20, If different from Report) S. KEY WORDS (Continue on reverse side if necessary and identify by block number) Onlinear Schrödinger equation DAAG29-75-C-0024 II. DAAG29-75-C-0024 III. PROGRAM ELEMENT PROJECT, TASK Work Unit Number 1 - Applied Analysis V. REPORT DATE Septembers 49 79 III. NUMBER OF PAGES T. SECURITY CLASS, (of this report) UNCLASSIFIED The Continue of this Report) UNCLASSIFIED The Continue on reverse side if necessary and identify by block number) Onlinear Schrödinger equation Distribution inequality DAAG29-75-C-0024 III. PROGRAM ELEMENT PROJECT, TASK Work Unit Number 1 - Applied Analysis UNC Unit Number 1 - Applied Analysis UNC REPORT DATE Septembers 49 79 III. Number of PAGES V. REPORT DATE Septembers 49 79 III. Number of PAGES V. REPORT DATE Septembers 49 79 III. Number of PAGES V. REPORT DATE Septembers 49 79 III. Number of PAGES V. REPORT DATE Septembers 49 79 III. Number of PAGES V. REPORT DATE Septembers 49 79 III. Number of PAGES V. REPORT DATE Septembers 49 79 III. Number of PAGES V. REPORT DATE Septembers 49 79 III. Number of PAGES V. REPORT DATE Septembers 49 79 III. Number of PAGES V. REPORT DATE Septembers 49 79 III. Number of PAGES V. REPORT DATE Septembers 49 79 III. Number of PAGES V. REPORT DATE Septembers 49 79 III. Number of PAGES V. REPORT DATE Septembers 49 79 III. Number of PAGES V. REPORT DATE Septembers 49 79 III. Number of PAGES V. REPORT DATE Septembers 49 79 III. Number of PAGES ARCA NUMBER OF RECEAUTION OF RECEAUTION OF RECEAUTION OF RECEAUTION OF RECEAUTION OF RECEAUT	NONLINEAR SCHRODINGER EVOLUTION EQUATIONS		reporting period
DAAG29-75-C-0024 Performing organization name and address Mathematics Research Center, University of Madison, Wisconsin 53706 Controlling office Name and address U.S. Army Research Office P.O. Box 12211 Research Triangle Park, North Carolina 27709 Monitoring Igency name a address/il different from Controlling Office) Approved for public release; distribution unlimited. DAAG29-75-C-0024 10. Pergorna Le Leurit Provider Trask Work Unit Number 1 - Applied Analysis Work Unit Number 1 - Applied Analysis 12. Report Date September 1979 13. Number of Pages 7 15. SECURITY CLASS. (at this report) UNCLASSIFIED 15e. DECLASSIFIED 15e. DECLASSIFIED 15e. DECLASSIFICATION/DOWNGRADING 8 DISTRIBUTION STATEMENT (at this Report) Approved for public release; distribution unlimited. 12. 12. 12. 12. 12. 12. 12. 12. 12. 12.	Commence of the second	- I want	6. PERFORMING ORG. REPORT NUMBER
DAAG29-75-C-0024 Performing organization name and address Mathematics Research Center, University of Madison, Wisconsin 53706 Controlling office Name and address U.S. Army Research Office P.O. Box 12211 Research Triangle Park, North Carolina 27709 Monitoring Igency name a address/il different from Controlling Office) Approved for public release; distribution unlimited. DAAG29-75-C-0024 10. Pergorna Le Leurit Provider Trask Work Unit Number 1 - Applied Analysis Work Unit Number 1 - Applied Analysis 12. Report Date September 1979 13. Number of Pages 7 15. SECURITY CLASS. (at this report) UNCLASSIFIED 15e. DECLASSIFIED 15e. DECLASSIFIED 15e. DECLASSIFICATION/DOWNGRADING 8 DISTRIBUTION STATEMENT (at this Report) Approved for public release; distribution unlimited. 12. 12. 12. 12. 12. 12. 12. 12. 12. 12.			
Performing Organization name and address Mathematics Research Center, University of Mathematics Research Center of Papers Moral University of Mathematics Research Center of Papers Moral University of Mathematics Research Vision University of Mathematics Research Center of Papers Moral University of Mathematics Research Center of Papers Moral University of Moral Univer	(Author(*)		8. CONTRACT OR GRANT NUMBER(8)
Performing Organization name and address Mathematics Research Center, University of Mathematics Research Center of Papers Moral University of Mathematics Research Center of Papers Moral University of Mathematics Research Vision University of Mathematics Research Center of Papers Moral University of Mathematics Research Center of Papers Moral University of Moral Univer	H./Brezis and T./Gallouet		DAAG29-75-C-0024
Mathematics Research Center, University of Madison, Wisconsin Madison, Wisconsin 53706 Madison, Wisconsin 53706 M. CONTROLLING OFFICE NAME AND ADDRESS U.S. Army Research Office P.O. Box 12211 Research Triangle Park, North Carolina 27709 MONITORING IGENCY NAME & ADDRESS(III different from Controlling Office) MONITORING IGENCY NAME & ADDRESS(III different from Controlling Office) Mapproved for public release; distribution unlimited. Mapproved for public release; distribution unlimited. MARCTSR1992 MEY WORDS (Contigue on reverse side if necessary and identify by block number) Monitoring Office		(1	S BARGEY-13-C-4021
Mathematics Research Center, University of Madison, Wisconsin Madison, Wisconsin 53706 Madison, Wisconsin 53706 M. CONTROLLING OFFICE NAME AND ADDRESS U.S. Army Research Office P.O. Box 12211 Research Triangle Park, North Carolina 27709 MONITORING IGENCY NAME & ADDRESS(III different from Controlling Office) MONITORING IGENCY NAME & ADDRESS(III different from Controlling Office) Mapproved for public release; distribution unlimited. Mapproved for public release; distribution unlimited. MARCTSR1992 MEY WORDS (Contigue on reverse side if necessary and identify by block number) Monitoring Office	PERFORMING ORGANIZATION NAME AND ADD	DRESS	10. PROGRAM ELEMENT, PROJECT, TASK
Madison, Wisconsin 53706 1. Contributing Office NAME AND ADDRESS U. S. Army Research Office P.O. Box 12211 Research Triangle Park, North Carolina 27709 13. Number of Process Research Triangle Park, North Carolina 27709 14. Monitoring Gency Name a Address(if different from Controlling Office) 15. SECURITY CLASS. (of this report) UNCLASSIFIED 15. DECLASSIFICATION DOWNGRADING SCHEDULE The Declassification of Downgrading Schedule Approved for public release; distribution unlimited. 12. 12. 12. 12. 12. 12. 12. 12. 12. 12.	Mathematics Research Center,	University of	
Madison, Wisconsin 53706 I. Contributing Office Name and Address U. S. Army Research Office P.O. Box 12211 Research Triangle Park, North Carolina 27709 A MONITORING IGENCY NAME & ADDRESS(II different from Controlling Office) D. Technical Summary Cept 15. SECURITY CLASS. (of this report) UNCLASSIFIED 15. DECLASSIFICATION/DOWNGRADING SCHEDULE TO DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited. D. DISTRIBUTION STATEMENT (of the abstract entered in Black 20, If different from Report) 8. Supplementary notes 9. KEY WORDS (Continue on reverse side if necessary and identify by block number) onlinear Schrodinger equation lobal solutions obolev embedding nterpolation inequality 10. ABSTRACT (Continue on reverse side II necessary and identify by block number) We consider the nonlinear Schrodinger equation	610 Walnut Street	Wisconsin	
A CONTROLLING OFFICE NAME AND ADDRESS U. S. Army Research Office P. O. Box 12211 Research Triangle Park, North Carolina 27709 4. MONITORING IGENCY NAME & ADDRESS(II different from Controlling Office) 13. NUMBER OF PAGES 7. MONITORING IGENCY NAME & ADDRESS(II different from Controlling Office) 15. SECURITY CLASS. (of this report) UNCLASSIFIED 15. DECLASSIFICATION/DOWNGRADING 8. DISTR BUTION STATEMENT (of this Report) Approved for public release; distribution unlimited. 12. DISTRIBUTION STATEMENT (of the abstract entered in Black 20, II different from Report) 8. SUPPLEMENTARY NOTES 9. KEY WORDS (Continue on reverse side if necessary and identify by block number) onlinear Schrödinger equation lobal solutions bobolev embedding interpolation inequality 10. ABSTRACT (Continue on reverse side if necessary and identify by block number) We consider the nonlinear Schrödinger equation	Madison, Wisconsin 53706		Applied Analysis
Research Triangle Park, North Carolina 27709 4. MONITORING GENCY NAME & ADDRESS(II different from Controlling Office) 15. SECURITY CLASS. (of this report) UNCLASSIFIED 15. DECLASSIFICATION/DOWNGRADING SCHEOULE 15. DESTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited. 12/12 7. DISTRIBUTION STATEMENT (of the abstract entered in Black 20, If different from Report) 8. SUPPLEMENTARY NOTES 8. SUPPLEMENTARY NOTES 9. KEY WORDS (Continue on reverse side if necessary and identify by block number) conclinear Schrodinger equation lobal solutions obsolev embedding nterpolation inequality 10. ABSTRACT (Continue on reverse side If necessary and Identify by block number) We consider the nonlinear Schrödinger equation	II. CONTROLLING OFFICE NAME AND ADDRESS	(1)	Management and an arrangement of the state of
Research Triangle Park, North Carolina 27709 4. MONITORING IGENCY NAME & ADDRESS(II dillerent from Controlling Office) 15. SECURITY CLASS. (of this report) UNCLASSIFIED 15. DECLASSIFICATION/DOWNGRADING SCHEDULE 15. DECLASSIFICATION/DOWNGRADING SCHEDULE 15. DECLASSIFICATION/DOWNGRADING 15. DECLASSIFICATION/DOWNGRADING SCHEDULE 16. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, II different from Report) 7. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, II different from Report) 8. SUPPLEMENTARY NOTES 16. NEY WORDS (Continue on reverse side if necessary and identify by block number) 17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, II different from Report) 18. SUPPLEMENTARY NOTES 19. KEY WORDS (Continue on reverse side if necessary and identify by block number) 19. ABSTRACT (Continue on reverse side if necessary and identify by block number) 19. ABSTRACT (Continue on reverse side if necessary and identify by block number) We consider the nonlinear Schrödinger equation		(11	
MONITORING IGENCY NAME & ADDRESS(II dillerent from Controlling Office) Technical Summary rept. UNCLASSIFIED 15a. DECLASSIFICATION/DOWNGRADING SCHEOULE Approved for public release; distribution unlimited. THE ADDRESS (In this Report) Approved for public release; distribution unlimited. THE ADDRESS (In this Report) DISTRIBUTION STATEMENT (of the abstract entered in Block 20, II different from Report) 8. SUPPLEMENTARY NOTES 8. SUPPLEMENTARY NOTES 8. SUPPLEMENTARY NOTES 9. KEY WORDS (Continue on reverse side if necessary and identify by block number) onlinear Schrodinger equation lobal solutions obsolev embedding interpolation inequality 10. ABSTRACT (Continue on reverse side if necessary and identify by block number) We consider the nonlinear Schrödinger equation		7a-1i-a 27700	13. NUMBER OF PAGES
UNCLASSIFIED 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE 15a. DECLASSIFICATION/DOWNGRADING 15a. DECLASSIFICATION/	Research Triangle Park, North (drollna 2//U9	15. SECURITY CLASS. (at this report)
S. DISTR BUTION STATEMENT (of this Report) Approved for public release; distribution unlimited. 12/12 7. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report) 8. SUPPLEMENTARY NOTES 8. SUPPLEMENTARY NOTES 8. SUPPLEMENTARY NOTES 9. KEY WORDS (Continue on reverse side if necessary and identify by block number) onlinear Schrödinger equation lobal solutions obolev embedding interpolation inequality 9. ABSTRACT (Continue on reverse side if necessary and identify by block number) We consider the nonlinear Schrödinger equation	THE MONTH ON THE CARDINGS IN		
Approved for public release; distribution unlimited. 1212 1212 7. DISTRIBUTION STATEMENT (of the abstract entered in Black 20, if different from Report) 8. SUPPLEMENTARY NOTES 9. KEY WORDS (Continue on reverse side if necessary and identify by block number) onlinear Schrödinger equation lobal solutions obolev embedding nterpolation inequality 9. ABSTRACT (Continue on reverse side if necessary and identify by block number) We consider the nonlinear Schrödinger equation	M. Tahain I Summ	an roota 1	UNCLASSIFIED
Approved for public release; distribution unlimited. 12112 7. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report) 8. SUPPLEMENTARY NOTES 8. KEY WORDS (Continue on reverse side if necessary and identify by block number) onlinear Schrödinger equation lobal solutions obolev embedding interpolation inequality 9. ABSTRACT (Continue on reverse side if necessary and identify by block number) We consider the nonlinear Schrödinger equation	O leguines , south	9	154. DECLASSIFICATION DOWNGRADING
Approved for public release; distribution unlimited. (12)12 7. DISTRIBUTION STATEMENT (of the abstract entered in Black 20, if different from Report) 8. SUPPLEMENTARY NOTES 8. SUPPLEMENTARY NOTES 6. KEY WORDS (Continue on reverse side if necessary and identify by block number) conclinear Schrödinger equation clobal solutions obolev embedding nterpolation inequality 6. ABSTRACT (Continue on reverse side if necessary and identify by block number) We consider the nonlinear Schrödinger equation			SCHEDUCE
8. SUPPLEMENTARY NOTES 9. KEY WORDS (Continue on reverse side if necessary and identify by block number) onlinear Schrödinger equation lobal solutions obolev embedding nterpolation inequality 9. ABSTRACT (Continue on reverse side if necessary and identify by block number) We consider the nonlinear Schrödinger equation	(14)MRC-	TSR-1992	
O. KEY WORDS (Continue on reverse side if necessary and identify by block number) onlinear Schrodinger equation lobal solutions obolev embedding nterpolation inequality O. ABSTRACT (Continue on reverse side if necessary and identify by block number) We consider the nonlinear Schrödinger equation	17. DISTRIBUTION STATEMENT (of the abstract e	ntered in Block 20, if different fro	m Report)
O. KEY WORDS (Continue on reverse side if necessary and identify by block number) onlinear Schrodinger equation lobal solutions obolev embedding nterpolation inequality O. ABSTRACT (Continue on reverse side if necessary and identify by block number) We consider the nonlinear Schrödinger equation			
O. KEY WORDS (Continue on reverse side if necessary and identify by block number) onlinear Schrodinger equation lobal solutions obolev embedding nterpolation inequality O. ABSTRACT (Continue on reverse side if necessary and identify by block number) We consider the nonlinear Schrödinger equation			
O. KEY WORDS (Continue on reverse side if necessary and identify by block number) onlinear Schrodinger equation lobal solutions obolev embedding nterpolation inequality O. ABSTRACT (Continue on reverse side if necessary and identify by block number) We consider the nonlinear Schrödinger equation			
O. KEY WORDS (Continue on reverse side if necessary and identify by block number) onlinear Schrodinger equation lobal solutions obolev embedding nterpolation inequality O. ABSTRACT (Continue on reverse side if necessary and identify by block number) We consider the nonlinear Schrödinger equation			
onlinear Schrödinger equation lobal solutions obolev embedding nterpolation inequality D. ABSTRACT (Continue on reverse side it necessary and identity by block number) We consider the nonlinear Schrödinger equation	18. SUPPLEMENTARY NOTES		
onlinear Schrödinger equation lobal solutions obolev embedding nterpolation inequality D. ABSTRACT (Continue on reverse side it necessary and identity by block number) We consider the nonlinear Schrödinger equation			
onlinear Schrödinger equation lobal solutions obolev embedding nterpolation inequality D. ABSTRACT (Continue on reverse side it necessary and identity by block number) We consider the nonlinear Schrödinger equation			
onlinear Schrödinger equation lobal solutions obolev embedding nterpolation inequality D. ABSTRACT (Continue on reverse side it necessary and identity by block number) We consider the nonlinear Schrödinger equation			
lobal solutions obolev embedding nterpolation inequality D. ABSTRACT (Continue on reverse side it necessary and identity by block number) We consider the nonlinear Schrödinger equation		sary and identify by block number;	
obolev embedding nterpolation inequality D. ABSTRACT (Continue on reverse side it necessary and identity by block number) We consider the nonlinear Schrödinger equation			
nterpolation inequality D. ABSTRACT (Continue on reverse side it necessary and identity by block number) We consider the nonlinear Schrödinger equation			
We consider the nonlinear Schrödinger equation	oppores empedding		
We consider the nonlinear Schrödinger equation			
We consider the nonlinear Schrödinger equation			
	Interpolation inequality	ary and identify by block numbers	
$\begin{cases} i \frac{\partial u}{\partial t} - \Delta u + k u ^2 u = 0 & \text{in } \Omega \times [0, \infty) \\ u(x,t) = 0 & \text{in } \partial\Omega \times [0, \infty) \\ u(x,0) = u_0(x) & \text{in } \Omega \end{cases}$	Interpolation inequality On ABSTRACT (Continue on reverse eide II necess		
u(x,t) = 0	Interpolation inequality 20. ABSTRACT (Continue on reverse eide if necess We consider the nonlinear in	Schrödinger equation	
$ (u(x,0) = u_0(x) $ in Ω	Interpolation inequality 20. ABSTRACT (Continue on reverse eide if necess We consider the nonlinear in	Schrödinger equation	[0,∞)
	Interpolation inequality 20. ABSTRACT (Continue on reverse eide if necess We consider the nonlinear in	Schrödinger equation	[0,∞) × [0,∞)

DD 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

20. ABSTRACT - Cont'd.

where Ω is a bounded domain or an exterior domain of \mathbb{R}^2 . Such an equation has been extensively studied when $\Omega = \mathbb{R}^2$, but the methods do not apply if $\Omega \neq \mathbb{R}^2$. We prove that there exists a unique global smooth solution if $k \geq 0$ or if k < 0 and $|k| \int |u_0|^2 < 4$. The proof relies on a new interpolation-embedding inequality:

$$\left\| u \right\|_{L^{\infty}} \leq C \{1 + \sqrt{\log (1 + \left\| u \right\|_{H^2})} \} \quad \text{for every } u \in H^2 \quad \text{with } \left\| u \right\|_{H^1} \leq 1 \ .$$